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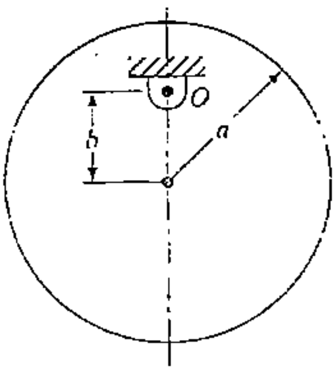
MEN 330
EXAM I
SPRING 2005

96 good

1) (20 pts) Find the natural frequency of oscillation of the system made up of a uniform circular disc pivoted at point O.

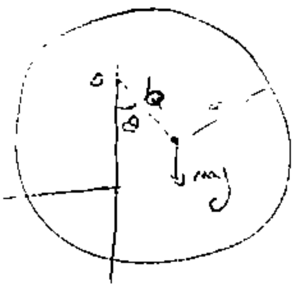


20
good

By Conservation of energy.

$U + T =$ constant.

$\Rightarrow I_O = \frac{1}{2} m a^2 + m b^2$



$\Rightarrow mgh + \frac{1}{2} I_o \dot{\theta}^2 =$ constant

then $\Rightarrow mg\,b(1-\cos\theta) + \frac{1}{2} I_o \dot{\theta}^2 =$ cte.

since θ is small $\Rightarrow$ $1-\cos\theta = \frac{\theta^2}{2}$

$\Rightarrow \frac{1}{2} mgb\,\theta^2 + \frac{1}{2} I_o \dot{\theta}^2 =$ cte.

Derive $\Rightarrow$ $mgb\,\theta\dot{\theta} + I_o \dot{\theta}\ddot{\theta} = 0$

$mgb\,\theta + \left(\frac{1}{2} m a^2 + m b^2\right)\ddot{\theta} = 0$ ✓

$\Rightarrow gb\,\theta + \left(\frac{1}{2} a^2 + b^2\right)\ddot{\theta} = 0$

$\Rightarrow \omega_n = \sqrt{\dfrac{gb}{\frac{1}{2}a^2 + b^2}}$ ✓

2) (15 pts) A machine weighing 2000 N rests on a support. The support deflects about 5 cm as a result of the machine. The floor under the support is somewhat flexible and moves, because of the motion of a nearby machine, harmonically at resonance with an amplitude of 0.2 cm. Assume a damping ratio of $\varsigma = 0.01$, and calculate the amplitude of the transmitted force and the amplitude of the transmitted displacement.

$m = \dfrac{2000}{9.81} = 203.87\ kg$.

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$F = Kx \Rightarrow K = \dfrac{F}{x} = \dfrac{2000}{5\times10^{-2}} = 40{,}000\ N/m$.

$\omega_n = \sqrt{\dfrac{K}{m}} = \sqrt{\dfrac{40000}{203.87}} = 14\ rad/s$

It is a base excitation problem

since resonance $\Rightarrow$ ~~[illegible]~~ $\omega_b = \omega_n \Rightarrow r = 1$

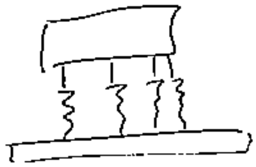
$$\frac{F_T}{KY} = r^2\left[\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}\right]^{1/2} \Rightarrow \frac{F_T}{40{,}000\times2\times10^{-3}} = \left[\frac{1+(2\times0.01)^2}{(2\times0.01)^2}\right]^{1/2}$$

$\Rightarrow \boxed{F_T = 4{,}000.8\ N.}$ = Amplitude of transmitted force

$$\frac{X}{Y} = \left[\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}\right]^{1/2} = \left[\frac{1+(2\times0.01)^2}{(2\times0.01)^2}\right]^{1/2}.$$

$\Rightarrow \underline{X = 0.1\ m.}$ Amplitude of transmitted displacement.

a) (15 pts) An electric motor weighing 750 *lb* and running at 1800 *rpm* is supported on four steel helical springs, each of which has eight active coils with a wire diameter of 0.25 *in* and a coil diameter of 3 *in*. The rotor has a weight of 100 *lb* with its center of mass located at a distance of 0.01 *in* from the axis of rotation. Find the amplitude of the force transmitted to the base of the motor.



$e = 0.01$ in

$m_0 = 0.2588$ slug

$\omega_r = 188.5$ rad/s

$m = 1.941$ slug

$$K = \frac{G d^4}{64 n R^3} = \frac{11\times10^6 \times 0.25^4}{64\times 8\times 1.5^3} = 24.866 \text{ lb/in} \checkmark \quad 13$$

$$K_{eq} = 4(24.866) = 99.464 \text{ lb/in} \checkmark$$ (since springs are in parallel).

Rotation Unbalance:

$$\Rightarrow \cancel{\frac{m_0 e}{m}} \quad \omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{99.464}{1.941}} = 7.16 \text{ rad/s}$$

$$r = \frac{\omega_r}{\omega_n} = 26.33 \checkmark$$

$$X = \frac{m_0 e}{m} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\xi r)^2}} \checkmark$$ $\xi = 0$ since no damping.

$$\Rightarrow X = \frac{0.2588\times 0.01}{1.941} \times \frac{26.33^2}{\sqrt{(1-26.33^2)^2}} = 1.33\times10^{-3} \text{ in} \checkmark$$

$$\frac{F_T}{KY} = r^2\left[\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2}\right]^{1/2} = 26.33^2\left[\frac{1}{(1-r^2)^2}\right]^{1/2} \times$$

$$\Rightarrow \frac{F_T}{99.464\times1.33\times10^{-3}} = 0.99855 \Rightarrow F_T = 0.132 \text{ lb} \checkmark$$

This transmitted force is due to unbalance, it should be added to the force done by weight

$\Rightarrow F_T = 0.132 + 750 = 750.132$ lb. $\times$